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Chapter 2

The Disappearance of Space and Time

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Abstract

I argue that lesson of general relativity is that at our present state of knowledge the best way for making sense of the world is to discard the notions of space and time. Newtonian space and time can be reinterpreted as aspects of the gravitational field, which is only one among the various dynamical physical fields making up the world. Physical fields do not need to inhabit spacetime in order to exist. The resulting understanding of space is to some extent a return to the Aristotelian-Cartesian relational tradition; while the resulting interpretation of temporality, appears to have strong elements of novelty. I consider the viability of a foundation of our understanding of the world in which space and time play no role.

1. The ontology of spacetime after relativity

Our understanding of the natural world evolves. We have developed a conceptual structure that allows us to apprehend and frame the world that we perceive and think; but this conceptual structure evolves, driven by experience and rational investigation. Science is a continuous exploration of novel and more effective ways for thinking the world. We cannot exit our own way of thinking; but we can modify it from within, exploring modifications of our basic assumptions, and testing them for consistence and against experience. This process of exploration of the space of the ideas is at the core of theoretical physics.

The notions of space and time of classical physics are a characteristic product of this process. We own them to a large extent to the work of Newton, in the 17th century. Newton defended a novel way of thinking space and time against the dominant views of his time. This way proved then extraordinarily effective. The Newtonian notions of space and time have been extensively utilized and discussed in depth. With time, they have been "absorbed" by our culture at large, and have become the dominant view.

The relativistic revolution of early 20th century, once more due to a remarkable extent to a single man, has taught us that there is a more effective way of understanding space and time than the Newtonian one. The novel *relativistic* understanding of space and time, however, has not yet been integrated into the common, and not even into the learned, way of thinking the world. Yes, mental habits take time to change; but I am often surprised by the excessive attachment that many thinkers maintain to ideas for understanding the world that have been clearly proven ineffective. These ideas were useful for a while, roughly in the three centuries between Newton and Einstein. But we must not mistake a tool that has proven useful for an eternal truth. There are commonly used concepts, such as the idea of an "objective present state of the world", that make no sense, in the light of what we have learned about the universe. Relativity is not "contemporary science": it is close to a century old. It is certainly time to take it seriously, discuss it in depth, and get used to it.

A reason for the slow adaptation to the relativistic understanding of space and time is that the relativistic revolution has happened in more than one step. The first step is special relativity (SR); shortly after came general relativity (GR)¹. For a few decades, while SR was blessed by continuous confirmations, especially from particle physics, GR had spectacular but scarce empirical support. In this situation, the attention was mostly on the relatively simpler conceptual novelty of SR, leaving GR in a limbo at the borders of our map of reality. But in the last decades the number and the success of experiments and observation confirming the physical validity of GR have exploded, and the theoretical interest in GR has boomed. Today, we cannot leave GR out of the picture. SR is little more than a minor variation of the Newtonian conceptualization of spacetime. The special relativistic universe is a theoretical model whose true interest as a fundamental way to understand reality was significant for less than 10 years, between 1905 and 1915. Therefore we must focus on GR if we want to hold a view of space and time compatible with what we have understood so far about the natural world.

¹To further complicate matters, the relativistic revolution has not yet ended, because we have not yet fully unraveled its relation with quantum physics. Knowledge of quantum theory induces us to certain conceptual choices in understanding relativity, but care should be taken in reading these hints because we still lack a definitive synthesis.

The new understanding of spacetime that has emerged from the relativistic revolution differs from the Newtonian picture especially with regard to the ontological status of spacetime — the subject of this book. Newton made the successful hypothesis that space and time are fixed structured background entities underlying material reality, which participate in governing the motion of physical objects. What Einstein has discovered is that Newton had mistaken a physical field for a background entity. The two entities hypostatized by Newton, space and time, are just a particular local configuration of a physical entity — the gravitational field — very similar to the electric and the magnetic field.

Einstein's discovery is that Newtonian space and time and the gravitational field are the same entity. There is a tradition of expressing this discovery saying that "there is no gravitational field: space and time become dynamical". I think that this is a convoluted and misleading way of thinking, which does not do justice to Einstein's discovery, and has the additional flaw of becoming meaningless as soon as we take into account the fact that the gravitational field has quantum properties.

The clean way of expressing Einstein's discovery is to say that there are no space and time: there are only dynamical objects. The world is made by dynamical fields. These do not live in, or on, spacetime: they form and exhaust reality.

One of these fields is the gravitational field. In the regimes in which we can disregard its dynamics, this field interacts with the rest of the physical objects as if it were a fixed background. This background is what Newton discovered and called space and time. We can keep using the evocative terminology "space-time" to indicate the gravitational field. But it has practically none of the features that characterized space and time. Relativistic spacetime is an entity far more akin to Maxwell's electric and magnetic fields than to Newtonian space.

In classical GR, a given solution of the field equations might still have some vague resemblance to the Newtonian's notions, since it defines a "continuum" which things can be imagined "to inhabit". But the only compelling reason for thinking that "spacetime" is the gravitational field, and not — say — the electromagnetic field, is the contingent fact that we live in a portion of the universe where the gravitational field is sufficiently constant for us to use it as a convenient reference.

Quantum mechanics reinforces this point of view. A solution of the classical field equations is like a particle trajectory: a notion that only makes physical sense in the classical limit. The gravitational field has quantum properties, and therefore it cannot define a spacetime continuum in the small.

Properly speaking, relativity has taught us that the effective way of thinking about the world in the light of what we have learned so far is to give up the notions of "space and time entities" entirely. This is not a dramatically radical

view, since it is not far from the way space was commonly conceptualized before Newton. On the other hand, it has a novel twist of great interest, especially as far as time, and the relation between time and space, are concerned.

In Newtonian physics, if we take away the dynamical entities, what remains is space and time. In relativistic physics, if we take away the dynamical entities, nothing remains. As Whitehead put it, we cannot say that we can have spacetime without dynamical entities, anymore than saying that we can have the cat's grin without the cat (Whitehead, 1983).

In the rest of this text, I discuss relativistic spacetime in some more detail. I start by recalling a few facts about pre-Newtonian western ideas about space and time. This is important because the Newtonian scheme is often mistaken for a sort of "natural" understanding of space and time. Nothing is more wrong: the Newtonian space and time "entities" form a strongly counter-intuitive theoretical construction, which met fierce resistance at first. Next, I illustrate the modification of the notions of spacetime introduced by SR and GR. I focus on the Newtonian notions that are to be abandoned. I close by mentioning the possibility of a proper relativistic foundation of the physical description of the world where the notions of space and time play no role.

2. Space

There are two traditional ways of understanding space in the western culture: as an *entity* or as a relation. "Space is an entity" means that space exists also when there is nothing else than space. It exists by itself, and objects may move in it. "Space is a relation" means that the world consists entirely of physical objects (particles, bodies, fluids, fields ...). These objects have the property that they can be in touch with one another, or not. Space is this "touch", or "contiguity", or "adjacency" relation between objects. Connected to these two manners of understanding space, are two manners of understanding motion. If space is an entity, motion can be defined as going from one part of space to another part of space. This is denoted by "absolute motion". If space is a relation, motion can only be defined as going from the contiguity of one object to the contiguity of another object. This is called "relative motion". For a physicist, the issue is which of these two ways of thinking about space and motion allows a more effective description of the world.

The dominant view in the European tradition, from Aristotle to Descartes, was to understand space and motion as relational. Aristotle, for instance, defines the spatial location of an object as "the inner surface of the innermost object that surrounds the body" (Aristotle, Physics, Book IV, Chapter 4[20], Aristotle, 1952). This is relational space. Descartes defines motion as "the

transference of one part of matter or of one body, from the vicinity of those bodies immediately contiguous to it, and considered at rest, into the vicinity of some others" (Descartes, *Principia Philosophiae*, Section II-25, p. 51, Descartes, 1983). Aristotle as well insists that motion is relative. He illustrates the point with the example of a man walking over a boat. The man moves with respect to the boat, which moves with respect to the water of the river, which moves with respect to the ground

The alternative view of space, as an independent entity, existed since ancient times (mostly in the Democritean tradition), but became dominant only with Newton. It is also the way spacetime (rather than space) is understood in SR. For Newton, space is absolute and motion is absolute: "So, it is necessary that the definition of places, and hence local motion, be referred to some motionless thing such as extension alone or *space*, in so far as space is seen truly distinct from moving bodies" (Newton, *De gravitatione et Aequipondio Fluidorum*, pp. 89–156, Newton, 1962). This is in patent contrast with Descartes definition, given above.

It should be noted that the difference between the two points of view is not so strong as it seems at first sight. Starting from a relational point of view, we can always *choose* a physical entity, refer all motion to this preferred entity and *call* this entity "space". Newton does not miss this point, and in fact he specifies that what is to be called space has to be "truly distinct from moving bodies". Newton thought he had discovered this entity "truly distinct from moving bodies", the way to detect it and its effects. GR is the realization that the entity discovered by Newton is not at all "truly distinct from moving bodies". In fact, it is barely distinguishable from the other fields.

In introducing the idea of absolute space, Newton did not challenge a long tradition with light heart: he devotes a long initial section of the *Principia* to explain the reasons for his choice. Today we can say that the strongest argument in Newton's favor is a posteriori: his theoretical construction works extraordinary well. Relational Cartesian and Leibnizian proposals were never as effective. But this was not Newton's argument. Newton invokes empirical evidence, discussing the famous bucket experiment. This experiment proves that there are physical effects (the bending of the surface of the water in the bucket) that do not depend on the relative motion of the water with respect to the surrounding objects (the bucket).

The surface of the water curves when the water rotates: but rotates with respect to what?

Newton argues that the only reasonable answer is absolute space. The concavity of the water's surface is an effect of the absolute circular motion of the water: the motion with respect to absolute space. This, claims Newton, proves the existence of absolute space. Newton's argument is subtle; it has been often

challenged, but it has withstood all attacks for three centuries². It has then collapsed under Einstein's alternative and more effective answer.

Newton needed (accelerated) motion, exemplified by the rotation of the water in the bucket, to be absolute for the foundation of his dynamics. Without this, Newton's main law $\vec{F} = m\vec{a}$ would not even make sense: what would be the meaning of the acceleration \vec{a} ?

Opposition to Newton's absolute space was very strong. Leibniz and his school argued fiercely against absolute motion and absolute acceleration. Doubts never really disappeared all along the centuries and a feeling kept lingering that something was wrong with Newton's argument. Ernst Mach returned to the issue suggesting that Newton's bucket argument could be wrong because the water does not rotate with respect to absolute space: it rotates with respect to the full matter content of the universe. But the immense empirical triumph of Newtonianism could not be overcome. For three centuries.

After three centuries, Einstein found a new and simpler answer. The bending of the surface of the water is due to the relative motion of the water with respect to a physical entity: the local gravitational field. It is the gravitational field, not Newton's inert absolute space that tells objects if they are accelerating or not, if they are rotating or not. There is no inert background entity such as Newtonian space: there are only dynamical physical entities. Among these are the fields, introduced in our physical picture of the world by Faraday and Maxwell. Among the fields is the gravitational field.

Whether the water surface in Newton's bucket is concave or flat is not determined by the motion of the water with respect to absolute space. It is determined by the physical interaction between the water and the gravitational field. Einstein's discovery is that Newton had mistaken the gravitational field for absolute space.

In Newtonian physics, the spacetime coordinates \vec{x} and t refer to absolute space. They can be identified with the reading of measuring devices carefully selected as the ones that capture the structure of space and time. This selection is obtained by monitoring and correcting the "inertial" effects such as the bending of the water of the water, which signal motion with respect to absolute space.

²Or course relationalism, i.e., the idea that motion can be defined only in relation to other objects, should not be confused with Galilean relativity. Galilean relativity is the statement that "rectilinear uniform motion" is a priori indistinguishable from stasis. This is equivalent to saying that velocity (just velocity!), is only relative to other bodies. Relationalism, on the other hand, holds that any motion (however zigzagging) is a priori indistinguishable from stasis. The very formulation of Galilean relativity assumes a nonrelational definition of motion: "rectilinear and uniform" with respect to what? When Newton claimed that motion with respect to absolute space is real and physical, he, in a sense, overdid it, by insisting that even rectilinear uniform motion is absolute. This caused a painful debate, because there are no physical effects of inertial motion (therefore the bucket argument fails for this particular class of motions). Newton is well aware of this point, which is clearly stated in the Corollary V5 of the Principia, but he chooses to ignore it in the introduction of the Principia. I think he did this just to simplify his argument, which was already hard enough for his contemporaries.

Einstein realized that finding out the pre-GR "inertial" \vec{x} and t is nothing else than detecting local features of the gravitational field. In the theoretical apparatus of GR, on the other hand, the spacetime coordinates \vec{x} and t have a completely different status, and it is only an unfortunate historical accident that they are denoted in the same manner as the pre-general-relativistic inertial coordinates. The relativistic \vec{x} and t coordinates have no direct physical meaning (unless we gauge fix them to represent something else). The reading of measuring devices is identified with quantities in the theory that are independent of \vec{x} and t.

More formally, in the mathematics of classical GR we employ a background "spacetime" manifold and describe the fields as living on this manifold. However, the diffeomorphism invariance of the theory demands that the localization on this manifold is pure gauge. That is, it is physically irrelevant. The manifold is just an artifice for describing a set of fields and other physical objects whose only "localization" is with respect to one another.

A state of the universe does not correspond to one field configuration over the spacetime manifold M. It corresponds to an equivalence class under active diffeomorphisms of field configurations. Therefore localization over M is physically irrelevant. In fact, M has no physical interpretation. It is a mathematical device without physical counterpart. It is a gauge artifact. M cannot be interpreted as a set of physical "events", or physical spacetime points "where" the fields take value. The only possibility of locating points is with respect to the dynamical fields and particles of the theory itself. It is meaningless to ask whether or not the gravitational field is flat around the spacetime point A, because there is no physical entity "spacetime point A". Contrary to Newton, spacetime points are not entities where particle and fields live³.

The gravitational field $g_{\mu\nu}(x)$ determines a four-dimensional continuum with a metric structure. Excessive significance is often attributed to this structure, as if distance was an essential property of space, or even an essential property of reality. This is like an Eskimo thinking that snow is an essential property of the ground.

We could have developed physics without ever thinking about distances, while nevertheless retaining the complete predictive and descriptive power of our theories. We live in physical conditions where atoms form and interact with the gravitational field in such a way that they maintain structures characterized by the fact that the integral of the gravitational field $d=\int \sqrt{g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}}$ along these structures is very stable. We call this integral d "distance", and we have developed useful mathematics — geometry — to describe the structure of these distances.

³Physical field are not attributes of space, anymore than a mosaic is an attribute of the wall. The wall can be taken away from the mosaic, without necessarilly destroying the mosaic. The clearest intuition of the nature of a field (in particular, a gauge field) is the original Faraday's intuition of a field as a collection of lines.

Geometry has repeatedly been mistaken for an a priori feature of reality. Euclidean geometry was erroneously thought of as necessary. Later, Riemannian geometry as well has been erroneously considered necessary. However, there is no a priori reason for which reality has to be understood as a continuum with metric properties. Nor, for that matter, as a continuum at all. Indeed, contemporary research in quantum gravity points in a very different direction.

Conceptually, what disappears with GR is the idea of space as the "container" of the physical world. As mentioned, this disappearance is not so revolutionary after all: to some extent it amounts to return to the pre-Newtonian view of space as a relation between equal-status physical entities.

Allow me to close this section with a playful observation on the relation between the shift in our views about space and our overall world conception. In the pre-Copernican world the cosmic organization of the "things" was hierarchically structured. The Heavens above, the Earth below, spheres in order of decreasing perfection. Objects were located with respect to one another — this served to grant each object a precise "status" in the grand scheme of things, analogous to the social position of humans. With the Copernican revolution, this hierarchical structure was lost. Position lost any ranking value. Newton offered reality a global frame. He offered every object the equal dignity of a position in a uniform space. For Newton this frame was grounded in God. He called space the "sensorium" of God: the world as perceived by God. Thus, the position in space for Newton is, literally, the position of the objects in the eyes of God. Against the multiplicity of the individual points of view determined by the observation of the relative motions, absolute space grants a single-organizing principle. According to Newton, our rationality allows us to unveil the absolute point of view of God (by detecting inertial effects such as the bending of the surface of the water). With or without such an explicit reference to God, for three centuries space has been regarded as the preferred Entity with respect to which all other entities are located. In the 20th and 21st centuries and with GR we have been learning that we do not need this frame to keep reality in place. Reality keeps itself in place. Objects interact with other objects, and this is reality. Reality is the net of these interactions. We do not need an external entity to hold this net. We do not need Space, to hold the universe. Maybe the Copernican revolution is finally being completed.

3. Time

The disappearance of physical time is the second characteristic feature of the relativistic revolution. The notion of time is harder to deal with than the notion of space, and represents a more radical step than the disappearance of space. Once again, much of the common understanding of time derives from the 17^{th} century. Galileo was the first to use a mathematical time variable t to formulate equations describing the motion of terrestrial objects. These are equations for functions X(t) of time. Newton was well aware that we never measure the variable t appearing in these equations directly. We use clocks whose reading T should be taken as a good approximation of the hypothetical "true" time t. As in the case of spatial measurements, we select better and better clocks by eliminating effects that the theory denounces as produced by the difference between T and t. The relation T(t), between clock reading and true time, can itself be calculated from the theory, using a mechanical model of the clock. From X(t) and T(t) we can compute X(T), which is the only relation we effectively observe. Newtonian theory is formulated in terms of the not directly observable quantity t. The scheme is delicate and involved, but it has worked wonderfully well for three centuries.

SR takes the first step out of the Newtonian understanding of time. SR does not change the Newtonian hypostatization of absolute space and time, but destroys the clean distinction between the two.

SR is the discovery that it makes no physical sense to say that two distant physical events happen "at the same time". It is true that Einstein provides a definition of simultaneity, two events A and B, relative to an object O in a given state of motion⁴. But although this is a useful working definition, it is a mistake to give it ontological significance. There is nothing in SR that would lead us to think that A and B have an ontic property of being "existant at the same time with respect to O", besides satisfying a useful conventional definition.

To illustrate this point, consider a standard expanding cosmological model. Its space like surfaces of homogeneity are formed by the events at equal proper time after the big bang, or equal Friedmann time $t_{\rm Fr}$; these are the surfaces naturally considered "simultaneous" in cosmology. These surfaces are *not* equal time surfaces according to Einstein's simultaneity definition⁵. Therefore, in a cosmological context we have the alternative to call either "simultaneous" events at the same Friedmann time, or events that satisfy Einstein's definition of simultaneity. Both definitions are useful. The choice between them is a matter of taste or computational convenience, not a matter of ontology.

The simple physical fact, revealed by SR, is that there are physical events on, say, Andromeda that have no temporal relation with events on Earth. A small gravitational wave passing in between could change Einstein's simultaneity between us and them by years, without affecting the physics here or there anymore

⁴The event A along the trajectory of an observer O is said to be simultaneous to a distant event B if a light signal emitted by O a time interval T before A and reflected by B, returns to O at a time T after A.

⁵I thank Marc Lachieze-Rey for pointing this out to me.

than works on the highway change relations between two cities. The lesson is that the idea that there exists a "now" all over the universe does not square with what we know about the universe. At best, we can talk of time lapsed along individual world lines, or time experienced by individual observers.

The picture of a Universe changing from one global instant to the next is incompatible with what we know about the world.

What is then "time" in the light of GR? This is a deep and important question that in my opinion has not yet been sufficiently investigated. I offer here what I think is the most useful answer to this question.

GR inherits from SR the melting of space and time into spacetime. Therefore the relational nature of space revealed by GR extends to time as well. It follows that in GR there is no background spacetime and therefore in particular no time along which things happen. GR teaches us that we must abandon the idea that the flow of time is an ultimate aspect of reality. The best description we can give of the world is not in terms of time evolution. The dynamics of GR itself cannot be cleanly described in terms of evolution in time.

There are many distinct notions of time employed in GR: coordinate time t, proper time S, clock times T, cosmological time $t_{\rm Fr}$, asymptotic Poincaré time The last two refer to the description of special solutions of the Einstein field equations only. They are irrelevant in a discussion of the ontology of time, because a different ontology for different solutions of the same theory is certainly unsatisfactory. Clock times are simply the readings of certain physical variables, which can be locally employed as the independent variable for convenience. Once again, they have nothing to tell us about the ontology of time. Coordinate time is unobservable (unless the gauge is fixed, in which case it designates something else) because of general coordinate invariance. The only residual time notion that keeps a resemblance of temporality is proper time. Proper time does not flow uniformly in the universe. It is defined along a world line and, generically, if two world lines meet twice, the two proper times lapsed between the two encounters differ. Proper time S depends on the gravitational field, which is influenced by the interaction with many systems. Typically, harmonic oscillations are isochronous in S. Therefore, S like the distance d described in the previous section, is just an observable feature of the gravitational field, which is particularly convenient to use as a stable reference in our environment, when describing the motion of objects assuming the gravitational field fixed. The dynamics of the gravitational field itself, on the other hand, cannot be naturally described in terms of evolution in any well-defined preferred time variable.

Instead, we must describe reality in terms of correlations between observables. We can measure physical quantities around us. The physical theory restricts the combinations of quantities that we can measure. It predicts relations between these quantities. Only in particular situations we can choose one quantity as the

independent variable, call it time, and express the others as functions of it. In general, this may not be possible, and the physical theory gives us constraints on the values of measurable quantities that we can obtain from physical measurements, not evolution laws in a preferred time variable. Quantum theory assigns probabilities to such outcomes.

Basic physics without time is viable, it is forced upon us by relativity, and it is conceptually coherent and consistent with our experience of the world. A complete discussion of the foundations of mechanics in the absence of a notion of time is given for instance in Rovelli (2002). Remarkably if we give up the idea that there is a special "time" observable, mechanics takes a far more compact and elegant form. This shift of point of view is forced upon us by classical GR. If, in addition, we take quantum theory into account, the spacetime continuum, with its last vague resemblance to temporality disappears completely, and we confront the absence of time at the fundamental level in full.

So, where does temporality, with all its peculiar features ("flow" of time, whatever this means, irreversibility, memory, awareness ...) come from? I think that all this has nothing to do with mechanics. It has to do with statistical mechanics, thermodynamics, perhaps psychology or biology. In Rovelli (1993) I have developed, in collaboration with Alain Connes, the idea that it may be possible to recover temporality from statistical mechanics, within an atemporal mechanical universe (statistical time hypothesis). If this point of view is correct, temporality is an artifact of our largely incomplete knowledge of the state of the world, not an ultimate property of reality.

Some people find the absence of time difficult to accept. I think that this is just a sort of nostalgia for the old Newtonian notion of an absolute "Time" along which everything flows, a notion already shown by SR to be inappropriate for understanding the real world. I think that the motivation for holding on to Poincaré invariance, to unitary time evolution, to the idea that there is a "Present" extending all over the universe, is only to provide an anchorage for our familiar notions, which are appropriate to describe the garden of our daily life. But a bit more at large, these are notions that are inappropriate to describe this beautiful and surprising world we inhabit.

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